Geometry: 7.1-7.3 Notes

NAME______________________

7.1 Angles of Polygons ____ **Date:**________________

Define Vocabulary:

diagonal

equilateral polygon

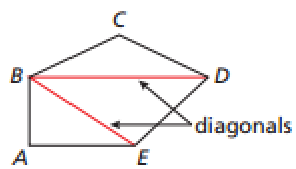
equiangular polygon

regular polygon

Using Interior Angles Measures of Polygons

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two nonconsecutive vertices.

Polygon *ABCDE*



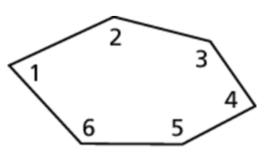
A and *B* are consecutive vertices.
Vertex *B* has two diagonals, $\overline{BD}$ and $\overline{BE}$.

Theorems

Theorem 7.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \bullet 180°$.

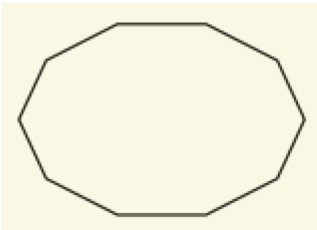
$$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \bullet 180°$$



$n = 6$

Examples: Find the sum of the measures of the interior angles.

<u>WE DO</u>



<u>YOU DO</u>

The coin shown is in the shape of an 11-gon

Examples: Find the number of sides of the polygon.

<u>WE DO</u>

The sum of the measures of the interior angles of a convex polygon is 1800°. Classify the polygon by the number of sides

<u>YOU DO</u>

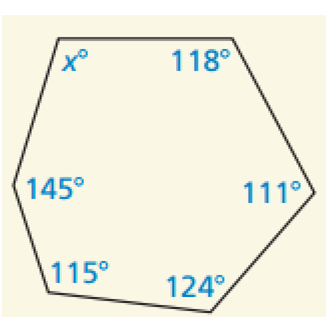
The sum of the measures of the interior angles of a convex polygon is 2520°. Classify the polygon by the number of sides

Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a quadrilateral is 360°.

Examples: Find the unknown interior angle measure.

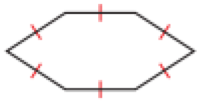
<u>WE DO</u>



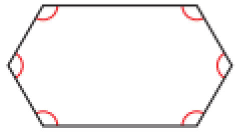
<u>YOU DO</u>

The measures of the interior angles of a quadrilateral are x°, $3x°, 5x°, and\ 7x°$. Find the measure of all the interior angles.

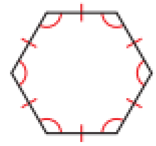
In an **equilateral polygon**, all sides are congruent.



In an **equiangular polygon**, all angles in the interior of the polygon are congruent.

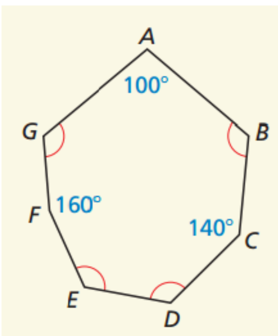


A **regular polygon** is a convex polygon that is both equilateral and equiangular.



Examples: Finding angle measures in polygons.

<u>WE DO</u>

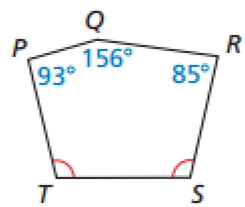


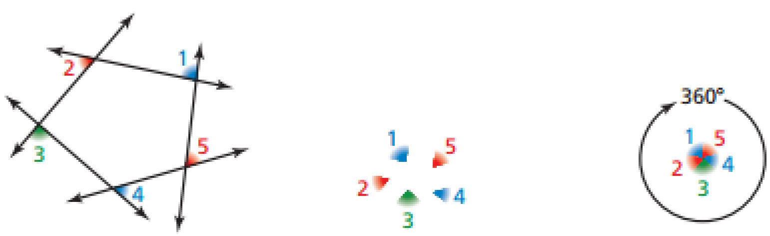
a. Is the polygon regular? Explain your reasoning.

b. Find the measures of $\angle B, \angle D, \angle E, and\ \angle G$.

<u>YOU DO</u>

Find the measures of $\angle S\ and\ \angle T$.



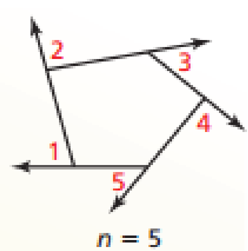


Theorem 7.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360^\circ$$

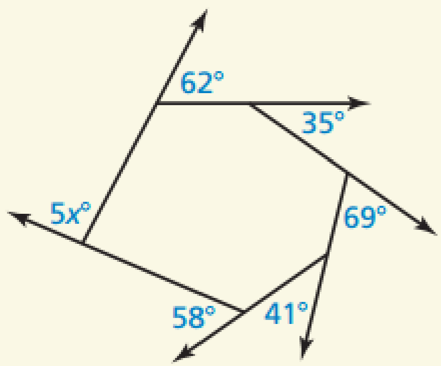
Proof Ex. 51, p. 366



Examples: Finding an unknown exterior angle measure.

<u>WE DO</u>

Find the value of x in the diagram.



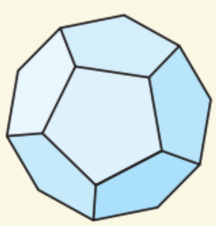
<u>YOU DO</u>

A convex hexagon has exterior angles with measures $34°, 49°, 58°, 67°, \textit{and}\ 75°$. What is the measure of an exterior angle at the 6th vertex?

Examples: Finding angle measure of regular polygons.

<u>WE DO</u>

Each face of the dodecahedron is shaped like a regular pentagon.



a. Find the measure of each interior angle of a regular pentagon.

b. Find the measure of each exterior angle of a regular pentagon.

<u>YOU DO</u>

a. Find the measure of each interior angle and each exterior angle of a regular 24-gon.

b. Each exterior angle of a regular polygon has a measure of 18°. Find the number of sides of the regular polygon.

Assignment	

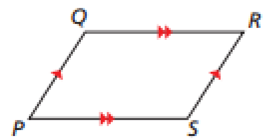
7.2 Properties of Parallelgrams___________________ _____________________________ Date:_____________

Define Vocabulary:

parallelogram

Using Properties of Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. In ▱$PQRS$, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.

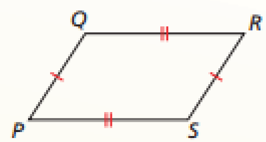


Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 368

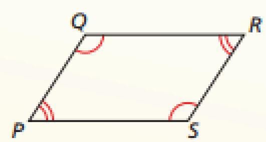


Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

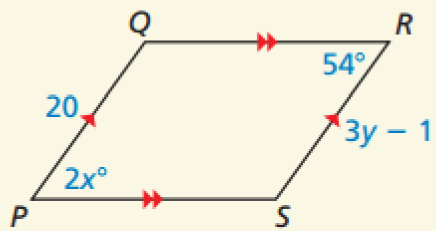
Proof Ex. 37, p. 373



Examples: Using properties of parallelograms.

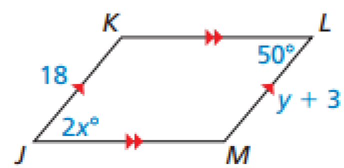
<u>WE DO</u>

Find the values of x and y.



<u>YOU DO</u>

Find the values of x and y.

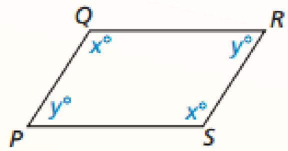


Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof Ex. 38, p. 373

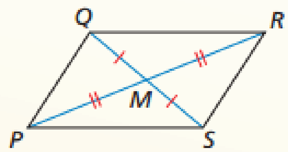


Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

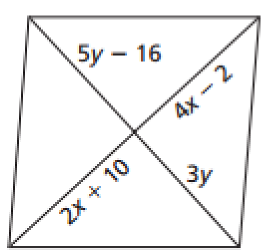
If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 370

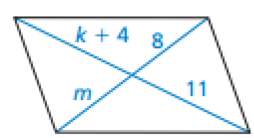


Examples: Find the value(s) of the variable(s) in the parallelogram.

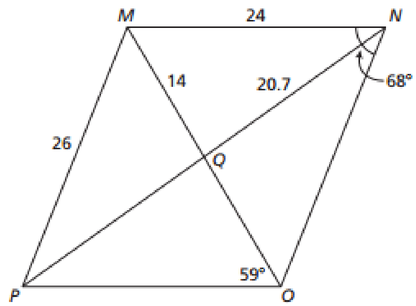
<u>WE DO</u>



<u>YOU DO</u>



Examples: Use the diagram to find the indicated measure.



WE DO

1. PO

2. OQ

3. $m\angle PMN$

4. $m\angle NOP$

YOU DO

1. NO

2. PQ

3. $m\angle OPM$

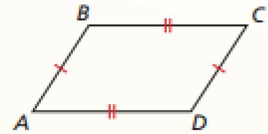
4. $m\angle NMO$

Assignment	

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

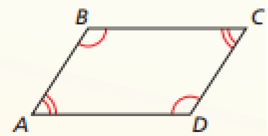


Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

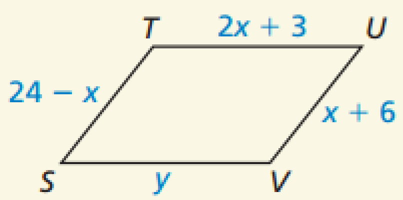
Proof Ex. 39, p. 383



Examples: Finding side lengths of a parallelogram.

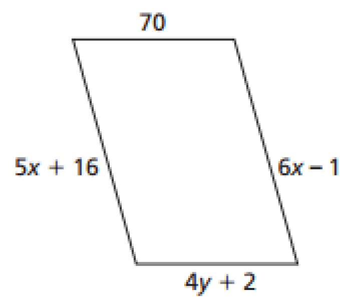
WE DO

For what values of x and y is quadrilateral $STUV$ a parallelogram?



YOU DO

Find the values of x and y that make the quadrilateral a parallelogram.

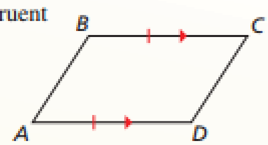


Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof Ex. 40, p. 383

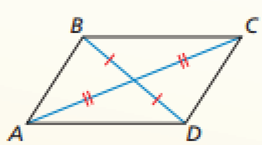


Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

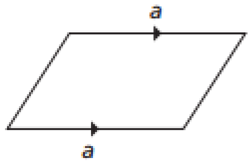
If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

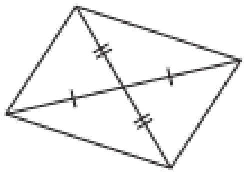
Proof Ex. 41, p. 383



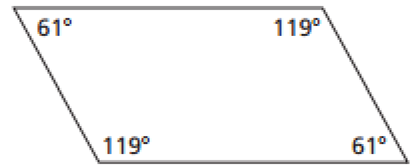
Examples: State which theorem you can use to show that the quadrilateral is a parallelogram.

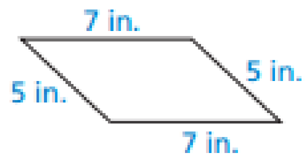
<u>WE DO</u>





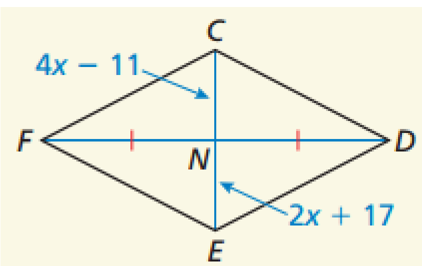
<u>YOU DO</u>



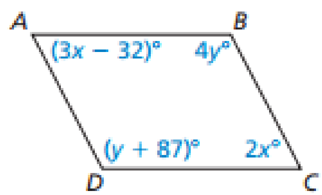


Examples: Find the value(s) of the variable(s) that make the quadrilateral a parallelogram.

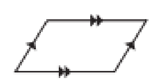
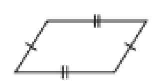
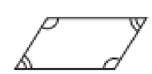
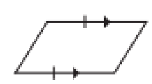
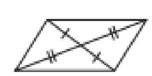
<u>WE DO</u>



<u>YOU DO</u>



Ways to Prove a Quadrilateral Is a Parallelogram

1. Show that both pairs of opposite sides are parallel. *(Definition)*	
2. Show that both pairs of opposite sides are congruent. *(Parallelogram Opposite Sides Converse)*	
3. Show that both pairs of opposite angles are congruent. *(Parallelogram Opposite Angles Converse)*	
4. Show that one pair of opposite sides are congruent and parallel. *(Opposite Sides Parallel and Congruent Theorem)*	
5. Show that the diagonals bisect each other. *(Parallelogram Diagonals Converse)*	

Assignment	